

Kinetics of polarization in non-relativistic scattering

A.I. Milstein* and S.G. Salnikov†

Budker Institute of Nuclear Physics, 630090 Novosibirsk, Russia

(Dated: March 15, 2013)

Abstract

An approach is developed, which essentially simplifies derivation of kinetic equation for polarization in non-relativistic scattering. This approach is applicable for collision of projectile particles with a target for any spins of colliding particles. The most detailed consideration is performed for the case of spin $1/2$ projectile particle interacting with spin $1/2$ particle of a target. The solution of the kinetic equation for the case of zero initial polarization is presented.

PACS numbers: 13.88.+e; 29.27.Hj

Keywords: Polarization; Kinetic equation; Scattering

*Electronic address: A.I.Milstein@inp.nsk.su

†Electronic address: salsergey@gmail.com

I. INTRODUCTION

It is known that the collision of high energy beams of polarized particles gives additional very important information as compared with the case of unpolarized particles. However, to obtain high energy beam of heavy particles with noticeable polarization is hard problem [1–3]. Besides, theoretical description of the kinetics of polarization is also nontrivial even in non-relativistic case and needs its further development, see Review [4]. In Ref. [5], the kinetics of the polarization buildup during the interaction of stored protons (antiprotons) with a polarized target was investigated. The kinetic equation was written in terms of the cross section corresponding to spin flip transition and spin non-flip transition. In Ref. [5], the quantization axes was directed along the polarization vector ζ_T of the target. If ζ_T is parallel or perpendicular to the momentum of particles in the beam, and the initial polarization of these particles is zero, then it follows from the arguments of parity that the polarization, arising as a result of interaction, is directed along ζ_T . Similar approach is used in Ref. [6] at the consideration of polarization effects in non-relativistic electron-proton scattering. If ζ_T is not parallel or perpendicular to the momentum of particles in the beam, then it is necessary to take into account additional terms in the kinetic equation which lead to rotation of polarization vector in the process of the polarization buildup [7]. The kinetic equation for the density matrix was derived for the first time in Ref. [8] by solving the quantum Liouville equation and expressing its solution via a spin-dependent scattering amplitude. The equation obtained in Ref. [8] was applied to the spin-exchange optical pumping, and it was shown that the spin-exchange collisions lead to rotation of electron spin. The same kinetic equation was later re-derived in Ref. [9] and applied to the analysis of the spin evolution, see also [10, 11].

In the present paper we develop an approach which essentially simplifies derivation of kinetic equation describing spin evolution for any spin of the target and particles in the beam. We consider the non-relativistic scattering and restrict ourselves for simplicity to the case of projectiles remaining in the beam after scattering. This case corresponds to small-angle scattering of the projectile. For instance, our approach can be directly applied to interaction of heavy particle with the flow of polarized electrons. However, our approach can be easily generalized to more complicated cases.

II. KINETIC EQUATION

Let us consider a particle with the spin S_1 interacting with a flow of polarized particles with the spin S_2 . Below this flow is referred to as a beam. We are going to investigate time evolution of $\langle \mathbf{S}_1 \rangle$. We assume for simplicity that the mass M_1 of the first particle is much larger than the mass M_2 of the second particle. In this case, in the rest frame of the first particle it is possible to consider the first particle as a source of some potential depending on the spin operators \mathbf{S}_1 and \mathbf{S}_2 . The operator \mathcal{O} is some operator constructed from the spin operators, so that $[\mathcal{O}, \mathbf{r}] = 0$ and $[\mathcal{O}, \mathbf{p}] = 0$, where \mathbf{r} is the relative coordinate vector and \mathbf{p} is the corresponding momentum. The wave function, which corresponds to the scattering problem, has the asymptotic form at large distances,

$$\psi_{\mathbf{k}}(\mathbf{r}) = \left[e^{i\mathbf{k} \cdot \mathbf{r}} + \frac{e^{ikr}}{r} F \right] \chi_1 \chi_2, \quad (1)$$

where \mathbf{k} is the initial momentum, χ_1 and χ_2 are the spin wave functions of the corresponding particles, the operator F depends on $\mathbf{n}_0 = \mathbf{k}/k$, $\mathbf{n} = \mathbf{r}/r$, and the spin operators. To describe kinetics of polarization, we should use the wave packet normalized to unity. We introduce the wave function

$$\Psi_{\mathbf{k}}(\mathbf{r}) = \frac{1}{\sqrt{V}} e^{-\lambda r} \psi_{\mathbf{k}}(\mathbf{r}), \quad (2)$$

where V is some normalization volume, and λ is some real parameter which we tend to zero at the end of calculations. The factor $e^{-\lambda r}$ allows one to perform integration by parts in the matrix elements (to use hermiticity of the operators), the system of units $\hbar = 1$ is used. Then we have the usual equation

$$\frac{d}{dt} \int d\mathbf{r} \Psi_{\mathbf{k}}^+(\mathbf{r}) \mathcal{O}_H \Psi_{\mathbf{k}}(\mathbf{r}) = iN \int d\mathbf{r} \Psi_{\mathbf{k}}^+(\mathbf{r}) [H, \mathcal{O}_H] \Psi_{\mathbf{k}}(\mathbf{r}), \quad (3)$$

where $\mathcal{O}_H = e^{iHt} \mathcal{O} e^{-iHt}$ is the Heisenberg operator, H is the Hamiltonian of the system, $N = 1/V$ is the density, and $[a, b]$ stands for the commutator of the operators a and b . Using the relation $H\psi_{\mathbf{k}}(\mathbf{r}) = E\psi_{\mathbf{k}}(\mathbf{r})$, we can write Eq. (3) as follows,

$$\begin{aligned} & \frac{d}{dt} \int d\mathbf{r} \Psi_{\mathbf{k}}^+(\mathbf{r}) \mathcal{O}_H \Psi_{\mathbf{k}}(\mathbf{r}) \\ &= iN \int d\mathbf{r} \psi_{\mathbf{k}}^+(\mathbf{r}) \{ [e^{-\lambda r}, H] \mathcal{O}_H e^{-\lambda r} + e^{-\lambda r} \mathcal{O}_H [e^{-\lambda r}, H] \} \psi_{\mathbf{k}}(\mathbf{r}). \end{aligned} \quad (4)$$

The commutator $[e^{-\lambda r}, H]$ is proportional to the small parameter λ , and only the contribution of large distances $r \sim 1/\lambda$ in the integral over \mathbf{r} can compensate this small parameter.

Therefore, at calculation of the commutator we can leave only the kinetic energy operator in the Hamiltonian,

$$[e^{-\lambda r}, H] \approx [e^{-\lambda r}, \frac{p^2}{2M}] = -\frac{i\lambda}{2M} (\mathbf{p} \cdot \mathbf{n} e^{-\lambda r} + e^{-\lambda r} \mathbf{n} \cdot \mathbf{p}) , \quad (5)$$

where $M \approx M_2$ is the reduced mass. We can also use the asymptotic form (1) of the wave function. Besides, in the term corresponding to interference of the plane wave and the spherical wave, the main contribution to the matrix element is given by the small angle θ between vectors \mathbf{n} and \mathbf{k} , namely $\theta^2 \sim 1/kr \sim \lambda/k$. Finally we obtain the kinetic equation

$$\frac{d}{dt} \langle \mathcal{O} \rangle = vN \text{Sp} \left\{ \rho(t) \left[\int d\Omega_{\mathbf{n}} F^+ \mathcal{O} F - \frac{2\pi i}{k} (F^+(0) \mathcal{O} - \mathcal{O} F(0)) \right] \right\} . \quad (6)$$

Here $v = k/M$, $d\Omega_{\mathbf{n}}$ is the differential of the solid angle corresponding to vector \mathbf{n} , $F(0)$ is the operator F calculated at $\mathbf{n} = \mathbf{n}_0$, and $\rho(t)$ is the density matrix which describes the spin state of the system, trace is taken over spin indexes of both particles. The density matrix equals to $\rho(t) = \rho_1(t)\rho_2$, where $\rho_1(t)$ is the time dependent density matrix of the first particle and ρ_2 is the time independent density matrix of the beam. The matrix $\rho_1(t)$ should be found as a result of solution of kinetic equation (see below). In (6) we assume that $F(0)$ is finite quantity, otherwise it is necessary to introduce a regularization.

If we set $\mathcal{O} = 1$ then we obtain the unitarity relation

$$\text{Sp} \left\{ \rho(t) \left[\int d\Omega_{\mathbf{n}} F^+ F - \frac{2\pi i}{k} (F^+(0) - F(0)) \right] \right\} = 0 . \quad (7)$$

Eqs. (6) and (7) are valid for arbitrary spins S_1 and S_2 of the particles.

III. THE CASE OF A PARTICLE WITH $S_1 = 1/2$.

Let $S_1 = 1/2$. Then it follows from Eq. (6) for $\mathcal{O} = \boldsymbol{\sigma}_1$ that

$$\begin{aligned} \frac{d}{dt} \boldsymbol{\zeta}_1 &= vN \text{Sp} \left\{ \rho(t) \left[\int d\Omega_{\mathbf{n}} F^+ \boldsymbol{\sigma}_1 F - \frac{2\pi i}{k} (F^+(0) \boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_1 F(0)) \right] \right\} , \\ \rho_1(t) &= \frac{1}{2} [1 + \boldsymbol{\zeta}_1(t) \cdot \boldsymbol{\sigma}_1] , \quad \boldsymbol{\zeta}_1(t) = \langle \boldsymbol{\sigma}_1 \rangle , \end{aligned} \quad (8)$$

where $\boldsymbol{\sigma}_1$ are the Pauli matrices acting on the spin variables of the first particle. The unitarity relation (7) gives

$$\begin{aligned} \text{Sp} \left[\rho_2 \int d\Omega_{\mathbf{n}} F^+ F \right] &= \text{Sp} \left[\frac{2\pi i}{k} \rho_2 (F^+(0) - F(0)) \right] , \\ \text{Sp} \left[\rho_2 \boldsymbol{\sigma}_1 \int d\Omega_{\mathbf{n}} F^+ F \right] &= \text{Sp} \left[\frac{2\pi i}{k} \rho_2 \boldsymbol{\sigma}_1 (F^+(0) - F(0)) \right] . \end{aligned} \quad (9)$$

For a particle with the initial polarization $\boldsymbol{\zeta}$ in a moment t , and the polarization $\boldsymbol{\zeta}_f$ measured by a detector, the cross section σ has the form

$$\begin{aligned}\sigma &= \text{Sp} \left[\rho(t) \int d\Omega_{\mathbf{n}} F^+ \rho_f F \right] = \frac{1}{2} (A + \mathbf{B} \cdot \boldsymbol{\zeta}_f), \\ \rho_f &= \frac{1}{2} [1 + \boldsymbol{\zeta}_f \cdot \boldsymbol{\sigma}_1], \\ A &= \text{Sp} \left[\rho(t) \int d\Omega_{\mathbf{n}} F^+ F \right], \quad \mathbf{B} = \text{Sp} \left[\rho(t) \int d\Omega_{\mathbf{n}} F^+ \boldsymbol{\sigma}_1 F \right].\end{aligned}\quad (10)$$

As a result of scattering the polarization becomes equal to $\boldsymbol{\zeta}' = \mathbf{B}/A$. Performing summation over $\boldsymbol{\zeta}_f$ and using the unitarity relation (7), we write the total cross section σ_{tot} as

$$\begin{aligned}\sigma_{tot} &= A = A_0 + \mathbf{B}_0 \cdot \boldsymbol{\zeta}_1, \\ A_0 &= \frac{\pi i}{k} \text{Sp} \left[\rho_2 (F^+(0) - F(0)) \right], \quad \mathbf{B}_0 = \frac{\pi i}{k} \text{Sp} \left[\rho_2 \boldsymbol{\sigma}_1 (F^+(0) - F(0)) \right].\end{aligned}\quad (11)$$

In terms of \mathbf{B} and \mathbf{B}_0 , Eq. (8) reads,

$$\frac{d}{dt} \boldsymbol{\zeta}_1 = vN \left\{ \mathbf{B} - \mathbf{B}_0 - \frac{\pi i}{k} \text{Sp} \left[\rho_2 (\boldsymbol{\zeta}_1 \cdot \boldsymbol{\sigma}_1) (F^+(0) \boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_1 F(0)) \right] \right\}, \quad (12)$$

It is convenient to write Eq. (8) also in another form,

$$\begin{aligned}\frac{d}{dt} \boldsymbol{\zeta}_1 &= vN \text{Sp} \left\{ \frac{1}{2} \rho(t) \int d\Omega_{\mathbf{n}} ([F^+, \boldsymbol{\sigma}_1] F + F^+ [\boldsymbol{\sigma}_1, F]) \right. \\ &\quad \left. + \frac{\pi}{k} \rho_2 [\boldsymbol{\zeta}_1 \times \boldsymbol{\sigma}_1] (F^+(0) + F(0)) \right\}.\end{aligned}\quad (13)$$

The quantity F can be written as

$$F = F_0 + \boldsymbol{\sigma}_1 \cdot \mathbf{F}_1, \quad (14)$$

where F_0 and F_1 are operators acting on the spin variables of the second particle. Then we obtain,

$$\begin{aligned}\frac{d}{dt} \boldsymbol{\zeta}_1 &= vN \text{Sp}_2 \left\{ \rho_2 \int d\Omega_{\mathbf{n}} [\mathbf{F}_1^+ (\mathbf{F}_1 \cdot \boldsymbol{\zeta}_1) + (\mathbf{F}_1^+ \cdot \boldsymbol{\zeta}_1) \mathbf{F}_1 - 2(\mathbf{F}_1^+ \cdot \mathbf{F}_1) \boldsymbol{\zeta}_1 \right. \\ &\quad \left. + i[(F_0^+ \mathbf{F}_1 - \mathbf{F}_1^+ F_0) \times \boldsymbol{\zeta}_1] - 2i[\mathbf{F}_1^+ \times \mathbf{F}_1] \right] - \frac{2\pi}{k} \rho_2 [(\mathbf{F}_1^+(0) + \mathbf{F}_1(0)) \times \boldsymbol{\zeta}_1] \left. \right\},\end{aligned}\quad (15)$$

where Sp_2 stands for trace over spin variables of particles from the beam (flow of polarized particles).

A. The case $S_2 = 0$.

For $S_2 = 0$ we have,

$$F = f_0 + f_1 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\nu}, \quad F_0 = f_0, \quad \mathbf{F}_1 = f_1 \boldsymbol{\nu}, \quad \boldsymbol{\nu} = [\mathbf{n} \times \mathbf{n}_0], \quad (16)$$

where f_0 and f_1 are some functions of $x = \mathbf{n} \cdot \mathbf{n}_0$. For $S_2 = 0$, the unitarity relations (9) reduces to one nontrivial relation

$$\int d\Omega_{\mathbf{n}} [|f_0|^2 + \nu^2 |f_1|^2] = \frac{4\pi}{k} \text{Im} f_0(0). \quad (17)$$

Using Eq. (15) we arrive at the following equation describing spin relaxation,

$$\begin{aligned} \frac{d}{dt} \boldsymbol{\zeta}_1 &= -\omega [\boldsymbol{\zeta}_1 + (\boldsymbol{\zeta}_1 \cdot \mathbf{n}_0) \mathbf{n}_0], \\ \omega &= vN \int d\Omega_{\mathbf{n}} \nu^2 |f_1|^2. \end{aligned} \quad (18)$$

The solution of this equation reads

$$\boldsymbol{\zeta}_1(t) = [\boldsymbol{\zeta}_1(0) - \mathbf{n}_0 (\boldsymbol{\zeta}_1(0) \cdot \mathbf{n}_0)] e^{-\omega t} + \mathbf{n}_0 (\boldsymbol{\zeta}_1(0) \cdot \mathbf{n}_0) e^{-2\omega t}. \quad (19)$$

Thus, during relaxation we have not only diminishing of $\zeta_1(t)$ but also rotation of the direction of $\boldsymbol{\zeta}_1(t)$.

B. The case $S_2 = 1/2$.

For $S_2 = 1/2$,

$$\begin{aligned} \rho_2 &= \frac{1}{2} [1 + \boldsymbol{\zeta}_2 \cdot \boldsymbol{\sigma}_2], \quad F = f_0 + (f_1 \boldsymbol{\sigma}_1 + f_2 \boldsymbol{\sigma}_2) \cdot \boldsymbol{\nu} + T^{ij} \sigma_1^i \sigma_2^j, \\ F_0 &= f_0 + f_2 \boldsymbol{\nu} \cdot \boldsymbol{\sigma}_2, \quad F_1^i = f_1 \nu^i + T^{ij} \sigma_2^j. \end{aligned} \quad (20)$$

Here $\boldsymbol{\sigma}_2$ are the Pauli matrices acting on the spin variables of the particle from the beam, $\boldsymbol{\zeta}_2$ is the time independent polarization of the beam. The functions, f_0 , f_1 , and f_2 depend on $x = \mathbf{n}_0 \cdot \mathbf{n}$, and the symmetric tensor T^{ij} is constructed from the vectors \mathbf{n}_0 and \mathbf{n} . Using these definitions, we obtain from Eq. (15),

$$\begin{aligned} \frac{d}{dt} \zeta_1^i &= R^{ij} \zeta_1^j + [\boldsymbol{\zeta}_1 \times \boldsymbol{\mathcal{F}}]^i + \mathcal{G}^i, \\ R^{ij} &= 2vN \int d\Omega_{\mathbf{n}} [|f_1|^2 (\nu^i \nu^j - \nu^2 \delta^{ij}) + \text{Re}(T^{ia*} T^{ja}) - T^{ab*} T^{ab} \delta^{ij}], \\ \mathcal{F}^i &= vN \left\{ 2 \int d\Omega_{\mathbf{n}} \text{Im} \left[f_0^* T^{ij} \zeta_2^j + f_2^* f_1 (\boldsymbol{\nu} \cdot \boldsymbol{\zeta}_2) \nu^i \right] + \frac{4\pi}{k} \text{Re} T^{ia}(0) \zeta_2^a \right\}, \\ \mathcal{G}^i &= 2vN \int d\Omega_{\mathbf{n}} \epsilon^{ijk} \epsilon^{abc} T^{ja*} T^{kb} \zeta_2^c. \end{aligned} \quad (21)$$

Here we use the relations,

$$\int d\Omega_{\mathbf{n}} \chi(\mathbf{n} \cdot \mathbf{n}_0) T^{ij} \nu^k = 0, \quad \int d\Omega_{\mathbf{n}} \epsilon^{abc} \text{Im}(T^{ia*} T^{jb}) = 0, \quad \int d\Omega_{\mathbf{n}} \text{Im}(T^{ia*} T^{ib}) = 0,$$

valid for any function $\chi(x)$. These relations can be easily proved using the representation of the tensor T^{ij} [12],

$$T^{ij} = \delta^{ij} f_3 + (n^i n^j + n_0^i n_0^j) f_4 + (n^i n_0^j + n_0^i n^j) f_5, \quad (22)$$

where $f_{3,4,5}$ are some functions of $\mathbf{n} \cdot \mathbf{n}_0$. For $S_2 = 1/2$, the unitarity relations (9) reduce to two nontrivial relations

$$\begin{aligned} \int d\Omega_{\mathbf{n}} [|f_0|^2 + \nu^2 |f_1|^2 + \nu^2 |f_2|^2 + T^{ab*} T^{ab}] &= \frac{4\pi}{k} \text{Im} f_0(0), \\ \int d\Omega_{\mathbf{n}} \left\{ 2\text{Re}(f_0^* T^{ab}) + 2\text{Re}(f_1^* f_2) \nu^a \nu^b - T^{ij*} T^{\alpha\beta} \epsilon^{i\alpha a} \epsilon^{j\beta b} \right\} &= \frac{4\pi}{k} \text{Im} T^{ab}(0). \end{aligned} \quad (23)$$

Then we obtain the form of the tensor R^{ij} and the vectors \mathcal{F} and \mathcal{G} ,

$$\begin{aligned} R^{ij} &= A_1 \delta^{ij} + B_1 n_0^i n_0^j, \\ \mathcal{F} &= A_2 \zeta_2 + B_2 (\zeta_2 \cdot \mathbf{n}_0) \mathbf{n}_0, \\ \mathcal{G} &= A_3 \zeta_2 + B_3 (\zeta_2 \cdot \mathbf{n}_0) \mathbf{n}_0, \end{aligned} \quad (24)$$

where A_i and B_i are some numbers. These numbers are expressed as

$$\begin{aligned} A_1 &= -vN \int d\Omega_{\mathbf{n}} [|f_1|^2 \nu^2 + T^{ia*} T^{ia} + n_0^i n_0^j T^{ia*} T^{ja}], \\ B_1 &= -vN \int d\Omega_{\mathbf{n}} [|f_1|^2 \nu^2 + T^{ia*} T^{ia} - 3n_0^i n_0^j T^{ia*} T^{ja}], \\ A_2 &= vN \left\{ \int d\Omega_{\mathbf{n}} \text{Im} [f_0^* (T^{ii} - n_0^i n_0^j T^{ij}) + f_2^* f_1 \nu^2] + \frac{2\pi}{k} \text{Re} [T^{ii}(0) - n_0^i n_0^j T^{ij}(0)] \right\}, \\ B_2 &= -vN \left\{ \int d\Omega_{\mathbf{n}} \text{Im} [f_0^* (T^{ii} - 3n_0^i n_0^j T^{ij}) + f_2^* f_1 \nu^2] + \frac{2\pi}{k} \text{Re} [T^{ii}(0) - 3n_0^i n_0^j T^{ij}(0)] \right\}, \\ A_3 &= -2vN \int d\Omega_{\mathbf{n}} [n_0^i n_0^j T^{ia*} T^{ja} - \text{Re}(n_0^i n_0^j T^{ij} T^{aa*})], \\ B_3 &= 2vN \int d\Omega_{\mathbf{n}} [3n_0^i n_0^j T^{ia*} T^{ja} - 3\text{Re}(n_0^i n_0^j T^{ij} T^{aa*}) + T^{ii} T^{jj*} - T^{ij} T^{ij*}]. \end{aligned} \quad (25)$$

Note that $A_1 < 0$ and $A_1 + B_1 < 0$.

If $\zeta_2 = 0$ then $\mathcal{F} = \mathcal{G} = 0$, and we obtain from Eq. (21)

$$\zeta_1(t) = [\zeta_1(0) - \mathbf{n}_0 (\zeta_1(0) \cdot \mathbf{n}_0)] e^{A_1 t} + \mathbf{n}_0 (\zeta_1(0) \cdot \mathbf{n}_0) e^{(A_1 + B_1)t}, \quad (26)$$

so that we have depolarization. The component of ζ_1 parallel to \mathbf{n}_0 and the component of ζ_1 transverse to this vector diminish with different rates.

Let $\zeta_2 \neq 0$ but $[\zeta_2 \times \mathbf{n}_0] = 0$ (ζ_2 is parallel to \mathbf{n}_0). In this case it is easy to find that

$$\begin{aligned}\zeta_1(t) &= e^{A_1 t} \{ \cos(\omega t) \zeta_1(0) + \sin(\omega t) [\zeta_1(0) \times \mathbf{n}_0] \} \\ &+ [e^{(A_1+B_1)t} - e^{A_1 t} \cos(\omega t)] (\zeta_1(0) \cdot \mathbf{n}_0) \mathbf{n}_0 \\ &+ [e^{(A_1+B_1)t} - 1] \frac{A_3 + B_3}{A_1 + B_1} \zeta_2, \quad \omega = (A_2 + B_2) \zeta_2.\end{aligned}\quad (27)$$

For $(\zeta_2 \cdot \mathbf{n}_0) = 0$ (ζ_2 is perpendicular to \mathbf{n}_0), the solution reads

$$\begin{aligned}\zeta_1(t) &= e^{(A_1+B_1/2)t} \cos(\Omega t) \zeta_1(0) \\ &+ \left\{ [e^{A_1 t} - e^{(A_1+B_1/2)t} \cos(\Omega t)] \frac{(\zeta_1(0) \cdot \zeta_2)}{\zeta_2^2} + [e^{A_1 t} - 1] \frac{A_3}{A_1} \right\} \zeta_2 \\ &+ e^{(A_1+B_1/2)t} \frac{\sin(\Omega t)}{\Omega} \left\{ A_2 [\zeta_1(0) \times \zeta_2] \right. \\ &\left. + \frac{B_1}{2} \left[(\zeta_1(0) \cdot \mathbf{n}_0) \mathbf{n}_0 - \frac{(\zeta_1(0) \cdot [\zeta_2 \times \mathbf{n}_0])}{\zeta_2^2} [\zeta_2 \times \mathbf{n}_0] \right] \right\}, \\ \Omega &= \sqrt{A_2^2 \zeta_2^2 - B_1^2/4}.\end{aligned}\quad (28)$$

For the general case, $[\zeta_2 \times \mathbf{n}_0] \neq 0$ and $(\zeta_2 \cdot \mathbf{n}_0) \neq 0$, it is convenient to write $\zeta_1(t)$ as

$$\zeta_1(t) = (\alpha + \gamma)(\zeta_2 \cdot \mathbf{n}_0) \mathbf{n}_0 - \beta(\zeta_2 \cdot \mathbf{n}_0) [\zeta_2 \times \mathbf{n}_0] - \gamma \zeta_2, \quad (29)$$

where α , β , and γ are some functions of time. For ζ_1^2 we have

$$\zeta_1^2(t) = \alpha^2 (\zeta_2 \cdot \mathbf{n}_0)^2 + \beta^2 (\zeta_2 \cdot \mathbf{n}_0)^2 [\zeta_2 \times \mathbf{n}_0]^2 + \gamma^2 [\zeta_2 \times \mathbf{n}_0]^2. \quad (30)$$

From Eqs. (21) and (24) we find

$$\begin{aligned}\frac{d\alpha}{dt} &= (A_1 + B_1)\alpha - A_2 [\zeta_2 \times \mathbf{n}_0]^2 \beta + A_3 + B_3 \\ \frac{d\beta}{dt} &= A_2 \alpha + A_1 \beta + (A_2 + B_2) \gamma \\ \frac{d\gamma}{dt} &= -(A_2 + B_2) (\zeta_2 \cdot \mathbf{n}_0)^2 \beta + A_1 \gamma - A_3.\end{aligned}\quad (31)$$

Let us write these equations in the matrix form,

$$\begin{aligned}\frac{d}{dt} \psi(t) &= U \psi + \xi, \\ \psi &= \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}, \quad \xi = \begin{pmatrix} A_3 + B_3 \\ 0 \\ -A_3 \end{pmatrix}.\end{aligned}\quad (32)$$

The formal solution of this equations has the form

$$\psi(t) = e^{Ut} [\psi(0) + U^{-1}\xi] - U^{-1}\xi. \quad (33)$$

The eigenvalues Λ_i of the matrix U read $\Lambda_i = A_1 - B_1\lambda_i$, where

$$\lambda_i^3 + \lambda_i^2 + (c_1 + c_2)\lambda_i + c_2 = 0, \quad c_1 = \frac{A_2^2}{B_1^2}[\zeta_2 \times \mathbf{n}_0]^2, \quad c_2 = \frac{(A_2 + B_2)^2}{B_1^2}(\zeta_2 \cdot \mathbf{n}_0)^2. \quad (34)$$

It is possible to show that all Λ_i have negative real parts so that the asymptotic form of $\psi(t)$ at large t is $\psi(t \rightarrow \infty) = -U^{-1}\xi$, or

$$\begin{aligned} \alpha &= -\frac{1}{W} \{ (A_2 + B_2)[(A_2 + B_2)(A_3 + B_3)(\zeta_2 \cdot \mathbf{n}_0)^2 + A_2 A_3 [\zeta_2 \times \mathbf{n}_0]^2] + A_1^2 (A_3 + B_3) \}, \\ \beta &= \frac{1}{W} [A_1(A_2 B_3 - A_3 B_2) - B_1 A_3 (A_2 + B_2)], \\ \gamma &= \frac{1}{W} [A_2(A_2 + B_2)(A_3 + B_3)(\zeta_2 \cdot \mathbf{n}_0)^2 + A_3 A_2^2 [\zeta_2 \times \mathbf{n}_0]^2 + A_1 A_3 (A_1 + B_1)], \\ W &= (A_1 + B_1)(A_2 + B_2)^2 (\zeta_2 \cdot \mathbf{n}_0)^2 + A_1 A_2^2 [\zeta_2 \times \mathbf{n}_0]^2 + A_1^2 (A_1 + B_1). \end{aligned} \quad (35)$$

As should be, this asymptotic form is independent of the initial condition. It is also time independent though $\zeta_1(t)$ changes its direction in a process of polarization at finite t .

IV. CONCLUSIONS

We have developed an approach which essentially simplifies derivation of the kinetic equation describing spin evolution for any spin of the target and particles in the beam. As an example, we have considered the non-relativistic scattering of the flow of polarized light particles on the heavy particle. For $S_1 = S_2 = 1/2$, we have obtained the explicit solution of the kinetic equation (21), which is expressed via a few constants, Eq. (25). The asymptotic form of the solution is independent of the initial condition and time (there is no rotation at large time) though $\zeta_1(t)$ changes its direction in a process of polarization. Our approach can be easily generalized to more complicated cases.

Acknowledgements

The work was supported by the Ministry of Education and Science of the Russian Federation.

- [1] Technical Proposal for Antiproton-Proton Scattering Experiments with Polarization, PAX Collaboration, arXiv:hep-ex/0505054 (2005).
- [2] Measurement of the Spin-Dependence of the \bar{p} -p Interaction at the AD-Ring, PAX Collaboration, arXiv:nucl-ex/0904.2325 (2009).
- [3] D. Oellers et al. , Phys.Lett. B **674**, 269 (2009).
- [4] X. Artru, M. Elchikh, J.-M. Richard, J. Soffer, O. V. Teryaev, Physics Reports **470**, 1 (2009).
- [5] A. I. Milstein and V. M. Strakhovenko, Phys. Rev. E **72**, 066503 (2005).
- [6] A.I. Milstein, S.G. Salnikov, V.M. Strakhovenko, Nucl. Instr. and Meth. B **266**, 3453 (2008).
- [7] N.N. Nikolaev and F.F. Pavlov, hep-ph/0601184.
- [8] L. C. Balling, R. J. Hanson, and F. M. Pipkin, Phys. Rev. **133**, A607 (1964).
- [9] V.G. Baryshevsky and A.G. Shekhtman, Phys. Rev. C **53**, 267 (19996).
- [10] V.G. Baryshevsky, "Optical" Spin Rotation Phenomenon and Spin Filtering of Antiproton (Proton, Deuteron) Beams in a Pseudomagnetic Field of a Polarized Target: the Possibility of Measuring the Real Part of the Coherent Zero-angle Scattering Amplitude, arXiv:1101.3146 .
- [11] V.G. Baryshevsky and A. R. Bartkevich, J. Phys. G: Nucl. Part. Phys. **39**, 125002 (2012).
- [12] L. Wolfenstein and J. Ashkin, Phys. Rev. **85**, 947 (1952).